

THE NEUTRON DISTRIBUTION IN NUCLEI AS MEASURED WITH PARITY VIOLATING ELECTRON SCATTERING

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A short review of the present knowledge of the nucleons distribution in nuclei is given. A proposal is made about a possible measurements of the neutron distribution through polarized electron scattering off nuclei.

1 The nuclear mean field

The cross sections involving neutrons and protons impinging on nuclei display a marked energy variation generally interpreted as an interference between the incident and the transmitted waves. This occurrence in turn implies a mean free path for collisions between the nuclear constituents large compared not only to the internucleonic distance, but even, sometimes, to the dimensions of the nucleus itself.

This finding is strongly suggestive of a mean field approach to the structure of the nucleus, especially as far as its ground state is concerned.

The nuclear mean field is implemented at the empirical level with the shell model. At the theoretical level a natural treatment resorts to the Hartree–Fock (HF) self-consistent theory, in a frame viewing nuclei as self-bound composite systems of nucleons interacting via a static two-body potential and governed by non-relativistic quantum mechanics.

Care is however required in handling the HF theory for the atomic nuclei. Indeed any realistic nucleon–nucleon (NN) force V_{NN} embodies so much repulsion that the expectation value of the nuclear Hamiltonian in the HF ground state is far from being enough attractive. In other words the HF wave function does not prevent two nucleons to come close to each other, where they experience a violent repulsion.

The Brueckner theory provides a remedy to this flaw. Indeed it yields an effective interaction G between two nucleons in a nucleus such that the interaction of two “uncorrelated” nucleons through G equals the interaction of two “correlated” nucleons through V_{NN} . Formally this result is achieved in the framework of perturbation theory by summing up (with the Bethe–Goldstone equation) the infinite set of ladder diagrams representing the scattering of two nucleons in the medium out of the Fermi sea, the other nucleons remaining passive.

Is the Brueckner–Hartree–Fock (BHF) accounting satisfactorily for the nuclear ground state properties? While the answer to this question is negative (the BHF mean field yielding a too large nuclear central density), yet an important lesson has been learned from BHF. It amounts to recognize that the perturbative corrections to the BHF mean field, necessary to reconcile theory and experiment, should not be computed by expanding in G , but rather by grouping together diagrams involving three, four, etc. nucleons repeatedly interacting among themselves through G (*hole line* expansion).

In spite of the fact that a proof of the convergence of the hole line expansion has never been provided, three- and four-hole line contributions have been actually computed in nuclear matter; however to achieve the same goal in finite nuclei has proved to be an almost impossible task.

Accordingly short cuts have been sought to incorporate the effects going beyond BHF still in a self-consistent mean field framework for the nuclear ground state.^b In this connection a successful approach allows for a density dependence of the interaction G , in addition to the one naturally induced by the Pauli operator in the Bethe–Goldstone equation. Indeed this procedure conveniently simulates the energy dependence and non-locality of G . Moreover it explains quite successfully the experimentally well established fact that in a nucleus the single particle orbits below the Fermi level are not 100% occupied as the HF (but not the BHF) approach would imply. Clearly the prize to be paid for the simplicity (and the success) is the introduction of a certain amount of phenomenology empirically fixing the local density dependence of G .

In the above outlined scheme, Negele¹ has been able to impressively reproduce over a wide range of momentum transfer the data of elastic electron scattering on several nuclei (like ^{16}O , ^{40}Ca and ^{208}Pb). This findings strongly support the view that most of the physics of the nuclear ground state lends indeed itself to be embodied in a “mean field”.

Should this be the case, then we would actually know and understand the nucleons’ distribution in nuclei. However, before drawing this conclusion, one should remind that electrons only probe the *proton* distribution. It would be therefore desirable to test the mean field scheme on other observables, the most natural one being the neutron distribution.

Although experimental information on the latter has been gathered in the past, e.g. with pion scattering on nuclei, still our knowledge of it remains rather poor. Even the recurring question of whether or not the nuclear surface is neutron rich cannot be presently answered with certainty.

^bWe do not mention, for brevity, relativistic mean field approaches which also look promising in reproducing the nuclear ground state properties.

Furthermore the present workshop stresses the urgency of reaching an accurate knowledge of the neutron distribution in order to achieve a precise interpretation of the atomic parity-violating (PV) experiments. Thus in the following we propose a method to measure the neutron distribution in the nuclear ground state, which is based on PV polarized electron-nucleus scattering.

2 The formalism of PV electron scattering

The helicity asymmetry, as measured in the scattering of right- and left-handed electrons off nuclei, is defined as follows

$$\begin{aligned}\mathcal{A} &= \frac{d^2\sigma^+ - d^2\sigma^-}{d^2\sigma^+ + d^2\sigma^-} \\ &= \mathcal{A}_0 \frac{v_L R_{AV}^L(q, \omega) + v_T R_{AV}^T(q, \omega) + v_{T'} R_{VA}^{T'}(q, \omega)}{v_L R^L(q, \omega) + v_T R^T(q, \omega)}\end{aligned}\quad (1)$$

where

$$v_L = \left(\frac{Q^2}{\bar{q}^2}\right)^2, \quad (2)$$

$$v_T = \frac{1}{2} \left| \frac{Q^2}{\bar{q}^2} \right| + \tan^2 \frac{\theta}{2} \quad (3)$$

and

$$v_{T'} = \sqrt{\left| \frac{Q^2}{\bar{q}^2} \right| + \tan^2 \frac{\theta}{2}} \tan \frac{\theta}{2} \quad (4)$$

are the usual lepton factors, θ is the electron scattering angle and $Q^2 = \omega^2 - \bar{q}^2 < 0$ is the space-like four momentum transfer of the vector boson carrying the electromagnetic (γ) or the weak neutral (Z_0) interaction.

In (1) the nuclear and nucleon's structure are embedded in both the parity conserving (electromagnetic) longitudinal and transverse (R^L and R^T) response functions and in the parity violating (weak neutral) ones. These classify as well in vector longitudinal and transverse (R_{AV}^L , R_{AV}^T) and axial transverse $R_{VA}^{T'}$ responses, the first (second) index in the subscript referring to the vector/axial nature of the weak neutral leptonic (hadronic) current. Finally the scale of the asymmetry is set by

$$\mathcal{A}_0 = \frac{\sqrt{2} G m_N^2}{\pi \alpha} \frac{|Q^2|}{4 m_N^2} \approx 6.5 \times 10^{-4} \tau \quad \left(\tau = \frac{|Q^2|}{4 m_N^2} \right) \quad (5)$$

in terms of the electromagnetic (α) and Fermi (G) coupling constants (m_N is the nucleon mass).

To gain informations on the structure of nuclei and nucleons the standard Coulomb, electric and magnetic multipole decomposition

$$R^L(q, \omega) = \sum_{J \geq 0} F_{CJ}^2(q) \quad (6)$$

and

$$R^T(q, \omega) = \sum_{J \geq 1} \{F_{EJ}^2(q) + F_{MJ}^2(q)\} \quad (7)$$

for the parity conserving (electromagnetic) and

$$R_{AV}^L(q, \omega) = a_A \sum_{J \geq 0} F_{CJ}(q) \tilde{F}_{CJ}(q) \quad (8)$$

$$R_{AV}^T(q, \omega) = a_A \sum_{J \geq 1} \{F_{EJ}(q) \tilde{F}_{EJ}(q) + F_{MJ}(q) \tilde{F}_{MJ}(q)\} \quad (9)$$

$$R_{VA}^{T'}(q, \omega) = -a_V \sum_{J \geq 1} \{F_{EJ}(q) \tilde{F}_{MJ_5}(q) + F_{MJ}(q) \tilde{F}_{EJ_5}(q)\} \quad (10)$$

for the parity violating (weak neutral) responses are performed.

In the above formulas ω is supposed to be given so the responses actually become functions of q only. Furthermore the Standard Model for the electroweak interaction is assumed: thus the vector and axial-vector leptonic coupling at the tree level read

$$a_V = 4 \sin^2 \theta_W - 1 \quad (11)$$

$$a_A = -1 \quad (12)$$

in terms of the Weinberg's angle. Finally the form factors, chosen to be real, with (without) a tilde are the weak-neutral (electromagnetic) ones. They, of course, split into isoscalar and isovector components according to the isospin decomposition

$$J_\mu = \xi^{(0)} (J_\mu)_{00} + \xi^{(1)} (J_\mu)_{10} \quad (13)$$

of the hadronic current.

Again, on the basis of the Standard Model at tree level, one has

$$\xi^{(0)} = \xi^{(1)} = 1 \quad (14)$$

in the electromagnetic sector. In the weak neutral sector, instead, one has

$$\xi^{(0)} = \beta_V^{(0)} = -2 \sin^2 \theta_W \simeq -0.461, \quad (15)$$

$$\xi^{(1)} = \beta_V^{(1)} = 1 - 2 \sin^2 \theta_W \simeq 0.538 \quad (16)$$

for the vector coupling and

$$\xi^{(0)} = \beta_A^{(0)} = 0, \quad \xi^{(1)} = \beta_A^{(1)} = 1 \quad (17)$$

for the axial one.

All the above formulas hold valid in the approximation of exchanging only one vector boson between the lepton and the hadron and up to a possible parity admixture in the nucleon itself (anapole moment of the nucleon) or in the nuclear states. The latter would stem from parity-violating components in the nucleon–nucleon interaction. These items will not be dealt with here. For a comprehensive treatment we refer the interested reader to the specialized literature on the subject.

3 Elastic polarized electron scattering from spin zero, isospin zero nuclei

As pointed out long time ago by Feinberg and Walecka² formula (1) applied to the elastic scattering of polarized electrons from a spin-zero, isospin-zero nuclear target leads to the simple expression

$$\mathcal{A} = \mathcal{A}_0 2 \sin^2 \theta_W \quad (18)$$

for the asymmetry.

It thus seemed that the opportunity was there to address the physics of the Standard Model in the low energy regime, being (18) a “model-independent” expression.

However things are not so simple because the isospin purity of the nuclear states, on which (18) relies, is not realized in nature: indeed the proton and neutron quantum states are different in a nucleus.

Accordingly, to account for the isospin breaking, (18) should be recast as follows

$$\begin{aligned} \mathcal{A} &= \mathcal{A}_0 a_A \beta_V^{(0)} \left\{ \frac{1 + \frac{\beta_V^{(1)}}{\beta_V^{(0)}} \frac{\langle J_i | \hat{M}_{0;10} | J_i \rangle}{\langle J_i | \hat{M}_{0;00} | J_i \rangle}}{1 + \frac{\langle J_i | \hat{M}_{0;10} | J_i \rangle}{\langle J_i | \hat{M}_{0;00} | J_i \rangle}} \right\} \\ &\simeq \mathcal{A}_0 2 \sin^2 \theta_W \left\{ 1 + \left(\frac{\beta_V^{(1)}}{\beta_V^{(0)}} - 1 \right) \frac{\langle J_i | \hat{M}_{0;10} | J_i \rangle}{\langle J_i | \hat{M}_{0;00} | J_i \rangle} \right\} \\ &= \mathcal{A}_0 2 \sin^2 \theta_W \left\{ 1 - \frac{1}{2 \sin^2 \theta_W} \frac{\langle J_i | \hat{M}_{0;10} | J_i \rangle}{\langle J_i | \hat{M}_{0;00} | J_i \rangle} \right\}. \end{aligned} \quad (19)$$

The above formula should of course be used “cum grano salis”. Indeed the expansion in the arguably small ground state matrix element of the *isovector* monopole operator $\hat{M}_{0;10}$ is warranted except where the ground state matrix element of the *isoscalar* monopole operator $\hat{M}_{0;00}$ is also or vanishing or very small, which happens, of course, at or close to the diffraction minima of the elastic cross-sections.

Thus, barring for these small domains, one would like to estimate, in the formula for the asymmetry

$$\mathcal{A} = \mathcal{A}_0 2 \sin^2 \theta_W [1 + \Gamma(q)] , \quad (20)$$

the impact of the nuclear dependent term $\Gamma(q)$, whose definition follows by comparing (19) with (20).

Before specifically addressing this issue, let us first answer the question: given that $\Gamma(q)$ is not zero, is it still small enough to render worth trying a measurement of $\sin^2 \theta_W$ with parity violating electron scattering?

Possibly this was the case a decade ago. Indeed around 1990 the value of the Weinberg’s angle quoted in the literature read

$$\sin^2 \theta_W = 0.227 \pm 0.005 , \quad (21)$$

namely it was given with a precision of 2.2%.

Accordingly in such a condition a meaningful determination of $\sin^2 \theta_W$ would have required on the one hand a measurement of the asymmetry to an accuracy of $1 \div 2\%$ and on the other to assume the isospin impurity in a nucleus to be below such a level.

Let us now see how these figures translate into a kinematical constraint. Given that an accuracy of 10^{-7} could be reached in measuring \mathcal{A} , a test of the Standard Model, as expressed by (21), would have had a good chance to be performed providing $\mathcal{A} \geq 10^{-5}$, in turn implying [from (5)] $q \geq 1.75 \text{ fm}^{-1}$. Now, always from (5), it follows that \mathcal{A} grows with q , but, at the same time, the elastic form factor falls off with q : one would thus choose the range

$$1.75 \leq q \leq 3.5 \text{ fm}^{-1} . \quad (22)$$

as a reasonable compromise between these two opposite requirements.

The question to be addressed was (and is) then: how large is $\Gamma(q)$ in the range (22) ? Is there $\Gamma(q) \leq 10^{-2}$? Whatever the answer to these questions might be, today the above arguing is of course untenable since now

$$\sin^2 \theta_W = 0.23055 \pm 0.00041 , \quad (23)$$

i.e, the Weinberg's angle is known with an accuracy of 0.18%.

Yet for others relevant observables, for example the strange parity content of the nucleon, although a generalization of the expression (20) for the asymmetry is required, a reliable theoretical handling of $\Gamma(q)$ is still crucial. This issue will be addressed in the next Section.

4 A simple model for isospin breaking

The isospin symmetry is broken in atomic nuclei by the Coulomb force which pushes the protons orbits outward with respect to the neutrons ones. On the other hand the strong proton–neutron interaction tries to equalize the proton's and neutron's Fermi energies, thus acting in the direction of restoring the isospin symmetry: the balance between these two effects is believed to leave a generally modest symmetry breaking in nuclei, which thus lends itself to a perturbative treatment.

To explore this physics Donnelly *et al.*³ worked out a simple model characterized by two monopole states ($J^\pi = 0^+$), one isoscalar and one isovector, mixed by the isospin breaking interaction, in particular the Coulomb one. In this scheme the dominantly isospin $T = T_0$ ground state is actually represented by the superposition

$$|“T_0”> = \cos \chi |T_0> + \sin \chi |T_0 + 1>, \quad (24)$$

and the dominantly $T_1 = T_0 + 1$ excited state by the orthogonal combination

$$|“T_1”> = -\sin \chi |T_0> + \cos \chi |T_0 + 1>. \quad (25)$$

The associated ground state matrix elements of the isoscalar and isovector monopole Coulomb operators read then, respectively,

$$\begin{aligned} < “T_0” | \hat{M}_{0;00}(q) | “T_0” > \\ = \cos^2 \chi < T_0 | \hat{M}_{0;00} | T_0 > + \sin^2 \chi < T_0 + 1 | \hat{M}_{0;00} | T_0 + 1 > \end{aligned} \quad (26)$$

and

$$\begin{aligned} < “T_0” | \hat{M}_{0;10}(q) | “T_0” > = \cos^2 \chi < T_0 | \hat{M}_{0;10} | T_0 > \\ + \sin^2 \chi < T_0 + 1 | \hat{M}_{0;10} | T_0 + 1 > + 2 \sin \chi \cos \chi < T_0 + 1 | \hat{M}_{0;10} | T_0 >. \end{aligned} \quad (27)$$

Let us then consider $N = Z$ nuclei. The Clebsch–Gordon (CG) coefficients entering into the reduction of the above matrix elements in isospace are 1 and $1/\sqrt{3}$, respectively, in formula (26), whereas in (27) only the third term survives and the associated CG is $1/\sqrt{3}$.

One thus obtains for the asymmetry in leading order of the mixing angle χ the expression

$$\begin{aligned} \mathcal{A} &= \mathcal{A}_0 a_A \frac{\beta_V^{(0)} \langle 0^+; 0 \| \hat{M}_{0;00} \| 0^+; 0 \rangle + \beta_V^{(1)} 2\chi \frac{1}{\sqrt{3}} \langle 0^+; 1 \| \hat{M}_{0;10} \| 0^+; 0 \rangle}{\langle 0^+; 0 \| \hat{M}_{0;00} \| 0^+; 0 \rangle + 2\chi \frac{1}{\sqrt{3}} \langle 0^+; 1 \| \hat{M}_{0;10} \| 0^+; 0 \rangle} \\ &\simeq \mathcal{A}_0 2 \sin^2 \theta_W \{1 + \Gamma(q)\} \end{aligned} \quad (28)$$

where

$$\Gamma(q) = 2 \left(\frac{\beta_V^{(1)}}{\beta_V^{(0)}} - 1 \right) \chi \mathcal{R}(q) = -\frac{1}{\sin^2 \theta_W} \chi \mathcal{R}(q) \quad (29)$$

and

$$\mathcal{R}(q) = \frac{1}{\sqrt{3}} \frac{\langle 0^+; 1 \| \hat{M}_{0;10} \| 0^+; 0 \rangle}{\langle 0^+; 0 \| \hat{M}_{0;00} \| 0^+; 0 \rangle} = \frac{F_{C0}(0^+; "1"; "0" \rightarrow 0^+)}{F_{C0}(0^+; "0"; "0" \rightarrow 0^+)}. \quad (30)$$

In the above the double bar matrix elements are meant to be reduced in isospace. Note that $\mathcal{R}(q)$ simply represents the ratio between the inelastic and the elastic form factors associated with the two monopoles states of our $N = Z$ nucleus.

Explicit calculations of $\Gamma(q)$ for a few nuclei have been performed by Donnelly *et al.*³ in a Wood–Saxon single particle wave functions basis with various effective interactions and in different configurations spaces. We display in Fig. 1a and 1b their results for $\Gamma(q)$ in C^{12} and Si^{28} . The singularities in the curves should be disregarded since they don't have, as previously discussed, any physical significance. Although the results of ref.³ are model dependent (they do change significantly according to the effective interaction employed), yet they convey the message that for light $N = Z$ nuclei like C^{12} the isospin breaking remains tiny indeed, below 1% over the whole range (22) of q .

Note also that in reaching this result the mixing angle χ has been extracted, with a quite conservative attitude, from the perturbative formula

$$\sin^2 \chi = \frac{|\langle T_0 + 1 | H_{CSV} | T_0 \rangle|^2}{(E_{T_0+1} - E_{T_0})^2} \quad (31)$$

(H_{CSV} is the charge symmetry violating part of the nuclear hamiltonian). In fact, while the experiments indicate for the matrix element appearing in the numerator of (31) a value ranging, in C^{12} , between 150 and 300 keV, the latter value has been adopted in obtaining Fig. 1a and 1b (moreover $\Delta E = E_{T_0+1} - E_{T_0} \simeq 17$ MeV according to the simple two levels model).

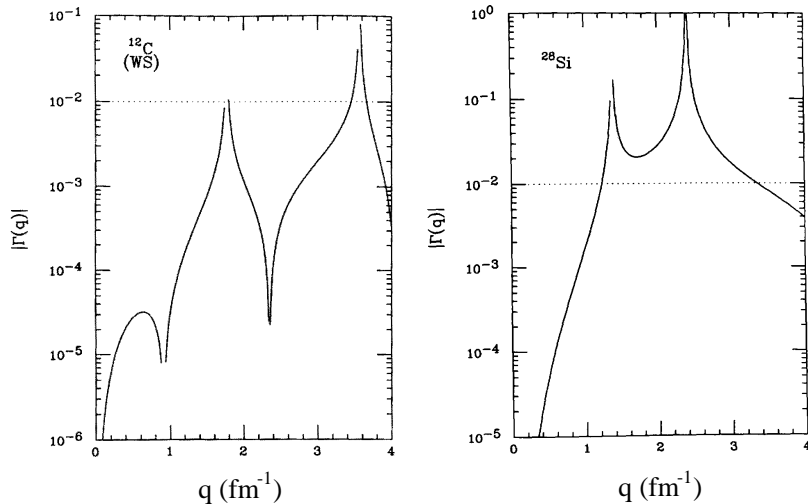


Figure 1: The nuclear-structure-dependent part of the parity-violating asymmetry as defined by eqs. (29) and (30). *Left:* calculations for elastic scattering from ^{12}C using Woods-Saxon single-particle wave functions. The dotted line at $|\Gamma(q)| = 10^{-2}$ indicates the level above which the structure-dependent effects would have confused the interpretation of the asymmetry as a test of electroweak theories, when these were known with the precision given by (21). *Right:* the same as in the left panel, but for ^{28}Si . For this nucleus Harmonic Oscillator single-particle wave functions and the shell model amplitudes as given by W.C. Haxton (unpublished) have been used.

These findings are supported by the results of a quite sophisticated calculation of Ramavataram *et al.*⁴. These authors, using a state of the art variational wave function originally due to Pandharipande *et al.*⁵, obtain in He^4 a $\Gamma(q)$ always well below 1% in the whole range (22). Of course this outcome also reflects the particularly rigid structure of He^4 , whose first excited state lies at 20.1 MeV (the first excited state of C^{12} is at 4.44 MeV).

It should however be pointed out that, for $N = Z$ but heavier nuclei like Si^{28} , the isospin breaking, while small, grows and reaches a few % in the range of q given by (22).

5 The case of $N \neq Z$ nuclei

For nuclei with $N \neq Z$ a novel (and important) feature appears. Although it will be illustrated in the specific case of nuclei with $N = Z + 2$, and therefore with third isospin component $M_T = -1$, it remains valid in all cases.

Sticking always to the two levels model⁸ we consider then a nucleus with a dominant isospin $T_0 = 1$ component in the ground state. Let in addition the nucleus have an excited state with a dominant isospin $T_1 = T_0 + 1 = 2$. Both states are further characterized by having $J^\pi = 0^+$.

Proceeding as in the case of a $Z = N$ nucleus, one arrives to an asymmetry still given by an expression like (28), but with the model dependent nuclear term $\Gamma(q)$ reading now as follows

$$\begin{aligned} \Gamma(q) = \frac{1}{2}\beta_V^{(1)} \left\{ -\frac{1}{\sqrt{6}} \langle 0^+; 1 \| \hat{M}_{0,1} \| 0^+; 1 \rangle + 2\sqrt{\frac{1}{10}}\chi \langle 0^+; 2 \| \hat{M}_{0,1} \| 0^+; 1 \rangle \right\} \\ \times \left\{ \sqrt{\frac{1}{3}} \langle 0^+; 1 \| \hat{M}_{0,0} \| 0^+; 1 \rangle - \frac{1}{\sqrt{6}} \langle 0^+; 1 \| \hat{M}_{0,1} \| 0^+; 1 \rangle + \right. \\ \left. + 2\sqrt{\frac{1}{10}}\chi \langle 0^+; 2 \| \hat{M}_{0,1} \| 0^+; 1 \rangle \right\}^{-1} \end{aligned} \quad (32)$$

(33)

which can again be recast according to

$$\Gamma(q) = \frac{1}{2}\beta_V^{(1)} \frac{F_{C0}(0^+; "1''; "1'' \rightarrow 0^+)_{\text{isovector}}}{F_{C0}(0^+; "1''; "1'' \rightarrow 0^+)_{\text{total}}}. \quad (34)$$

Namely $\Gamma(q)$ turns out to be, like before, proportional to the ratio between the inelastic (isovector) and the elastic (this time both isoscalar and isovector) form factors.

Now from (32) a new feature is immediately apparent: unlike the $N = Z$ case, where $\Gamma(q)$ was found to be proportional to the mixing parameter χ , here $\Gamma(q)$, in addition to terms proportional to χ , also embodies terms *independent* from it. As a consequence, $\Gamma(q)$ turns out to be now much larger, as it is clearly observed in Fig. 2a and 2b, where the results of ref.³ are displayed for C^{14} and Si^{30} .

Actually what is shown in the figures 2a and 2b is not $\Gamma(q)$, but $\tilde{\Gamma}(q)$ since the $N \neq Z$ nuclei are more appropriately discussed in terms of protons and neutrons rather than in term of isospin and, accordingly, the expression (20) for the asymmetry is now more conveniently recast into the form:

$$\mathcal{A} = \mathcal{A}_0 a_A \left(\beta_V^p + \frac{N}{Z} \beta_V^n \right) [1 + \tilde{\Gamma}(q)] \quad (35)$$

where

$$\beta_V^p = \frac{1}{2} \left(\beta_V^{(0)} + \beta_V^{(1)} \right) = 0.038, \quad (36)$$

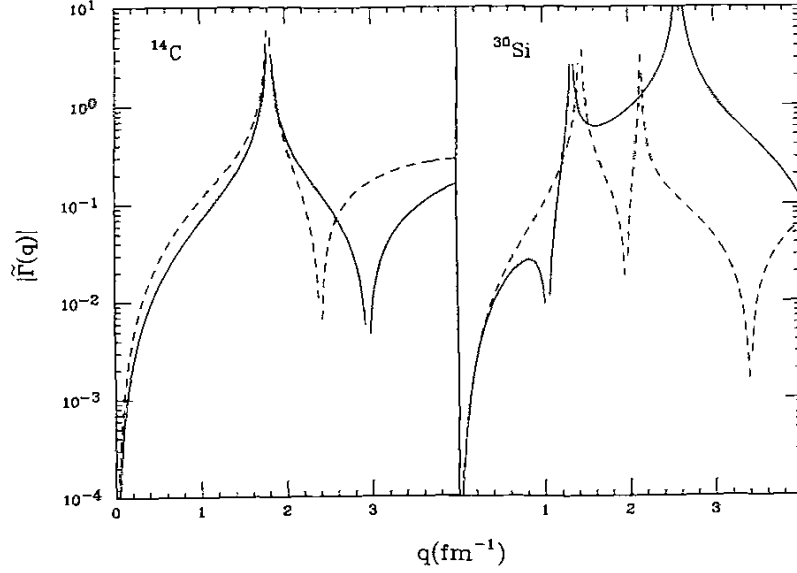


Figure 2: The structure-dependent part of the parity-violating asymmetry for elastic scattering as defined in eqs. (38) and (39). The results for ^{14}C using a 1p-shell model space (solid line) and a $2\hbar\omega$ -space (dashed line) are displayed. Also displayed are the results for ^{30}Si in the extreme single-particle shell model (solid line) and in a full 2s1d shell-model calculation (dashed line). No isospin mixing effects are included.

$$\beta_V^n = \frac{1}{2} \left(\beta_V^{(0)} - \beta_V^{(1)} \right) = -\frac{1}{2} \quad (37)$$

and

$$\tilde{\Gamma}(q) = \frac{1}{2} \tilde{\beta}'_V \frac{\langle 0^+ \left| \frac{1}{2} \left(1 - \frac{Z}{N} \right) \hat{M}_{0;00} + \frac{1}{2} \left(1 + \frac{Z}{N} \right) \hat{M}_{0;10} \right| 0^+ \rangle}{\langle 0^+ \left| \hat{M}_{0;00} + \hat{M}_{1;10} \right| 0^+ \rangle}, \quad (38)$$

being

$$\tilde{\beta}'_V = 4 \frac{N}{Z} \frac{\beta_V^n}{\beta_V^p + \frac{N}{Z} \beta_V^n}. \quad (39)$$

It is immediately checked that by setting $N = Z$ in the above formulas one gets $\tilde{\Gamma}(q) \rightarrow \Gamma(q)$ and $\tilde{\beta}'_V = 1/\sin^2 \theta_W$.

Let us further observe that, although in the figures 2a and 2b $\tilde{\Gamma}(q)$ appears indeed to be markedly model dependent, yet the basic message previously

anticipated clearly stands out: in the range of momentum transfers (22) $\tilde{\Gamma}(q)$ assumes values typically ranging between 10 and 50%.

In conclusion, from the previous two Sections it follows that for light $N = Z$ nuclei, like He^4 and C^{12} ,

- a test of the electroweak theory with parity-violating polarized elastic electron scattering experiments is out of question or, perhaps, only marginally, if at all, possible at very low momentum transfer in He^4 ;
- however many investigations show that elastic (and, also, quasielastic, not addressed here) parity violating polarized electron scattering experiments can be usefully exploited to unravel the strange form factor of the nucleon.

For medium-heavy and heavy nuclei $\Gamma(q)$ grows with the mass number A , especially when $N \neq Z$ (large terms not proportional to χ appear!). Therefore parity-violating polarized electron scattering can be advantageously used to measure the amount of isospin breaking in a nucleus or, better yet, the neutron distribution.

6 The neutron distribution

To appreciate how the neutron distribution can be measured with polarized elastic electron scattering it helps to revisit the previous concepts with a somewhat different language. Let us thus observe that for $T = 0$ nuclei ($N = Z$) if isospin is a “good symmetry” then the standard unpolarized electron scattering measures the *isoscalar* nuclear density. In these conditions the latter also fixes the polarized elastic electron scattering which is thus independent from any further nuclear structure information.

If, however, isospin is slightly broken, then the *isovector* nuclear density also enters, as a small perturbation, in the elastic polarized electron scattering thus introducing an additional model dependence.

Quite on the contrary, for $T \neq 0$ ($N \neq Z$) nuclei both the *isoscalar* and the *isovector* densities enter into the scattering process, no matter if isospin is or is not a perfect symmetry. In this instance, as already pointed out, it is preferable to use the neutron-proton language. Accordingly the ground state matrix element of the Coulomb monopole operator

$$\langle 0^+ | \hat{M}_0(q) | 0^+ \rangle = \frac{1}{\sqrt{4\pi}} \int d\vec{x} j_0(qx) \rho(\vec{x}), \quad (40)$$

where j_0 is the zeroth order spherical Bessel function and $\rho(\vec{x})$ is the matter

density of the nucleus, will display both an isoscalar and an isovector components, according to the expressions

$$\langle 0^+ | \hat{M}_{0;00}(q) | 0^+ \rangle = \frac{1}{\sqrt{4\pi}} \int d\vec{x} j_0(qx) \frac{\rho_p(\vec{x}) + \rho_n(\vec{x})}{2} \quad (41)$$

and

$$\langle 0^+ | \hat{M}_{1;00}(q) | 0^+ \rangle = \frac{1}{\sqrt{4\pi}} \int d\vec{x} j_0(qx) \frac{\rho_p(\vec{x}) - \rho_n(\vec{x})}{2} \quad (42)$$

In the above ρ_p and ρ_n are the protons and neutrons densities, respectively.

When (41) and (42) are inserted into the formula (35) it is then an easy matter to obtain for the asymmetry the expression^c

$$\mathcal{A} = \mathcal{A}_0 a_A \left\{ \beta_V^p + \beta_V^n \frac{\int d\vec{x} j_0(qx) \rho_n(\vec{x})}{\int d\vec{x} j_0(qx) \rho_p(\vec{x})} \right\} \quad (44)$$

Since, according to the Standard model,

$$\beta_V^p = 0.038 \quad \text{and} \quad \beta_V^n = -0.5, \quad (45)$$

(43) shows that *the asymmetry in the parity-violating elastic polarized electron scattering represents an almost direct measurement of the Fourier transform of the neutron density*, the analogous quantity for the protons being fixed by the elastic unpolarized electron scattering.

In particular a rigorous q -independence of $\mathcal{A}/\mathcal{A}_0$ would imply

$$Z\rho_n(\vec{x}) = N\rho_p(\vec{x}), \quad (46)$$

i.e. pure isospin symmetry! Of course, as previously discussed, in nuclei the distribution of neutrons differs from the one of protons.

To gain a first insight on how this difference would be perceived in a parity violating elastic electron scattering experiment Donnelly *et al.* have computed the asymmetry (44) and the $\tilde{\Gamma}(q)$ of eq. (43) for Ca^{40} , Ca^{48} and Pb^{208} using phenomenological proton densities which well accomplish for the

^cNote that, by comparison, (43) yields

$$\tilde{\Gamma}(q) = \frac{N}{Z} \frac{\beta_V^n}{\beta_V^p + \frac{N}{Z}\beta_V^n} \left[1 - \frac{Z \int d\vec{x} j_0(qx) \rho_n(\vec{x})}{N \int d\vec{x} j_0(qx) \rho_p(\vec{x})} \right]. \quad (43)$$

elastic unpolarized electron scattering. For example, for Ca^{40} they employ the well-known 3 parameters Fermi distribution:

$$\rho(\vec{r}) = \rho_0 \frac{1 + \omega \frac{r^2}{R^2}}{1 + e^{(r-R)/a}}. \quad (47)$$

For the neutrons they use the same densities, euristically enlarging however, for explorative purposes, the radius parameter by 0.2 fm. Their results are displayed in Fig. 3. To grasp the significance of this figure it helps to expand $\tilde{\Gamma}(q)$ as follows:

$$\tilde{\Gamma}(q) = \frac{\beta_V^n}{\beta_V^p + \frac{N}{Z}\beta_V^n} \left[\frac{N}{Z} - \frac{N - \frac{1}{6}q^2 R_n^2}{\int d\vec{x} j_0(qx) \rho_p(\vec{x})} + \dots \right]. \quad (48)$$

It is thus clear that the region around the first dip/peak carries information on the neutron radius whereas the fourth moment of the neutron distribution will be observed nearby the second dip/peak and so on. On the basis of this expansion one finds that in the case of Pb^{208} at $q \simeq 0.5 \text{ fm}^{-1}$, with $\mathcal{A} \simeq 8 \times 10^{-8}$, a 1% change in the neutron radius is reflected in a change of about 6% in the asymmetry.

It might be interesting to observe that formula (48) can be generalized in the sense that the Fourier transform of the pure Fermi distribution can be, although not exactly, quite accurately analytically expressed. Indeed for this density the form factor turns out to be⁶

$$F_{\text{Fermi}}(q) = \rho_0 (2\pi)^2 R^2 a \left\{ j_1(qR) \tilde{y}_0(\pi qa) - \tilde{j}_1(\pi qa) y_0(qR) \right\} \frac{\pi qa}{\tilde{j}_0(\pi qa)} \quad (49)$$

where the $j(y)$ and the $\tilde{j}(\tilde{y})$ are the “spherical” and the “modified spherical” Bessel functions of first (second) kind, respectively⁷.

The form factor F_3 of the three-parameters Fermi distribution is then expressed in terms of F_{Fermi} according to:

$$F_3(q) \simeq F_{\text{Fermi}}(q) - \frac{\omega}{R^2} \frac{d^2}{dq^2} [q F_{\text{Fermi}}(q)]. \quad (50)$$

When inserted into (44) and (43), the above formula allows to analytically recover the results of Donnelly for Ca^{40} .

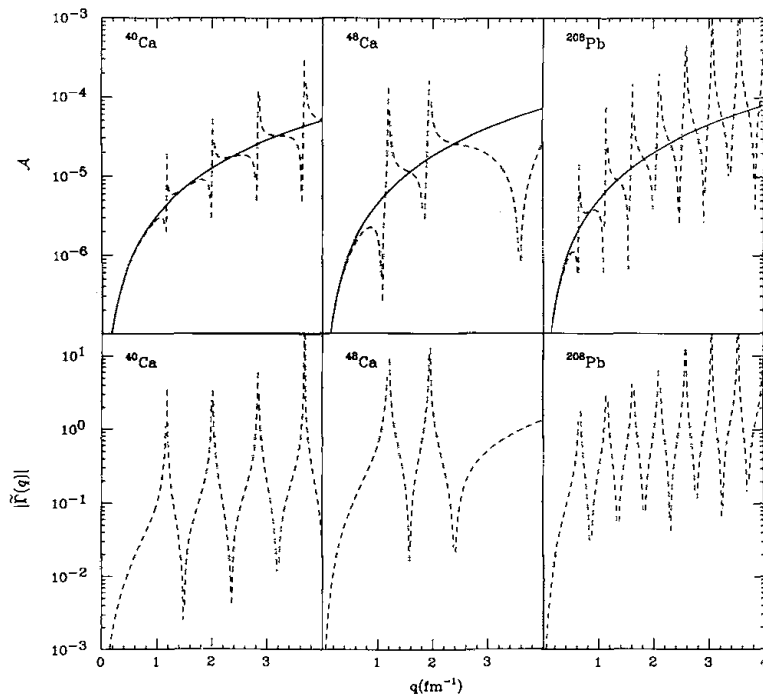


Figure 3: The parity-violating asymmetry (upper row) and the structure-dependent part of the asymmetry as defined in eq. (48) (lower row) for elastic electron scattering from ^{40}Ca , ^{48}Ca and ^{208}Pb . Displayed are the calculations with $\rho_n(r)/N = \rho_p(r)/Z$ [solid line, corresponding to $\tilde{\Gamma}(q) = 0$] and for $\rho_n(r)/N \neq \rho_p(r)/Z$ (dashed line). The density parameterizations are discussed in the text and specified in ref.³.

7 Flaws and hopes

The results of the previous section are flawed essentially by two shortcomings: one relates to the factorization of the single nucleon physics, which has been assumed in deducing (44) from (35). Especially when q is large, and thus relativistic effects become substantial, such a procedure almost certainly becomes unwarranted. It is clear that this point must be more carefully addressed in future research.

The second problem relates to the approximation of considering just the Fourier transform of the charge and neutron distributions, which corresponds to consider a single vector boson exchange between the impinging field and

the target, leaving the electron to be described by plane waves. This is clearly insufficient, especially in heavy nuclei, where, in fact, the electron wave is quite distorted by the nuclear Coulomb field. To account for this effect a heavy computational effort is presently carried out by an MIT–Indiana University collaboration. A more modest, but perhaps also useful approach, resorts to a kind of eikonal approximation to describe the distortion of the electron wave.

We feel confident that these difficulties will in the end be overcome. It will thus become possible to measure the neutron distribution in the ground state of atomic nuclei with parity violating *elastic* scattering experiments.

This most remarkable occurrence goes in parallel with the finding of Alberico *et al.*⁸ for the *quasi-elastic* polarized electron–nucleus scattering. Indeed these authors show that, being the Z_0 almost “blind” to protons, the PV longitudinal response of an uncorrelated system of protons and neutrons, like the relativistic Fermi gas, to a polarized beam of electrons is almost vanishing. Departure from this expectation will thus signal the effect of neutron–proton correlations in nuclei, which are expected to be especially relevant in the isoscalar channel.

Thus polarized electron scattering experiments appear to open a window on crucial, and till now insufficiently explored, aspects of the nuclear structure.

A final remark is in order: parity–violating *nuclear* electron scattering experiments, initially conceived as a tool for exploring the Standard Model at the nuclear level, will turn out, in the end, to represent a tool for providing the information the *atomic* parity violating experiments need to accurately test the Standard Model at the atomic level: namely the neutron distribution. Indeed the precision on the measured energy levels of the Cesium atoms, which is required to test the Standard Model, is so high that it cannot be reached without controlling also the neutron distribution in nuclei.

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